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A MODEL OF THE ENVIRONMENTALLY AFFECTED GROWTH OF FATIGUE CRACKS†

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The process of damage accumulation and crack growth under a combination of cyclic loads and environmental effects (the so-called corrosion fatigue) are considered. The equilibrium and energy balance relations in the "solid with cracks-load" system are supplemented by equations which describe the process of damage accumulation at the crack tips and their continuation. In addition to a phenomenological measure of the microdamage of mechanical origin, parameters characterizing the change in the properties near the contact surface due to physicochemical factors are introduced. The model takes into account the influence of extremal stresses of the cycle, the loading frequency and the parameters of the environment (such as the concentration of the corrosive agent or pH value), and the influence of the initial conditions on the rate of crack growth, the level of damage and the effective concentration ratio on the moving front, as well as the total time to fracture, measured both in number of cycles and in units of time. The model gives a satisfactory description of the behaviour of corrosion fatigue cracks.

Crack growth under the effect of cyclic and/or prolonged loads is a purely mechanical process that is studied in fracture mechanics (in its broad sense). The scale of the process varies widely, from the level of a crystal lattice to the level of structural elements. To construct models of the growth of fatigue cracks, therefore, in addition to classical continuum mechanics, one must use the phenomenological or structural models of the mechanics of the accumulation of dispersed damage. All the stages of crack growth, from nucleation to final fracture, can be described by a synthesis of fracture mechanics and the continuum mechanics of damage accumulation [1, 2].

Models of fatigue fracture become more complicated when the influence of the active surrounding media (liquid or gaseous) on the properties of the material and its state near the crack tip must be taken into account. A distinction is normally drawn between crack growth under the combined action of cyclic stresses and a surface-active agent, or corrosion fatigue, and corrosion cracking, in which the change of stress over time plays a secondary role. These phenomena are often both present, frequently showing up in the influence of moderate stresses on crack growth when a cyclic component is present.

Corrosion fatigue is a phenomenon in which mechanical, chemical, electrochemical and absorption processes interact. An example is the growth of fatigue cracks accompanied by the formation and fracture of an oxide film, in which crack growth is alternately slowed down and activated as newly revealed surfaces of the metal dissolve. A different mechanism is provided by crack propagation, accompanied by the diffusion of hydrogen near the crack tip and embrittlement of the metal near its end. Most investigations on corrosion fatigue have concerned the electrochemical, metallographic and chemical aspects of the phenomenon. The models of corrosion fatigue are, as a rule, qualitative in nature [3]. The prediction of crack growth is based on the principle of superposition or a modification of that principle. In the simplest case, the crack growth rate can be represented as the sum of two rates, one for a neutral medium while the other is a certain weighted value of the corrosion rate [4].

The growth of a macroscopic crack is the result of the interaction between the mechanisms of the accumulation of microdamage and the conditions of stability of the loaded body as a mechanical system. The main difficulty lies in determining the generalized forces which appear under equilibrium conditions and the stability of the "body with cracks-load" system. In the mechanics of brittle fracture, the generalized forces are taken to be such quantities as the intensity of energy release or the J-integral. In the mechanics of fatigue fracture, computing the generalized forces involves isochronal variation of the states of the "body with cracks-load" system, in which allowance must be made for the history of loading, deformation, microdamage accumulation and crack growth.

1. STATEMENT OF THE PROBLEM

We shall treat a body with cracks under an applied load as a mechanical system with unilateral

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constraints, to allow for the irreversibility of cracks in ordinary structural materials. As in [1, 2], we use Griffiths' concept of variation: in the isochronal transition to adjacent equilibrium states the only factors subject to variation are the crack parameters, while the equilibrium equations, the consistency of deformations, and all the boundary conditions, apart from the conditions at the crack tips, are satisfied. The crack parameters $a_1, \dots, a_m$ can then be interpreted as generalized coordinates. Their variations satisfy the conditions $\delta a_j \geqslant 0$ $(j = 1, \dots, m)$, by definition.

A system is called a "sub-equilibrium" system if the virtual work of all the external and internal forces calculated under Griffiths' variation $\delta W < 0$; it is called an "equilibrium" system if there are variations in which $\delta W = 0$, and for the other variations $\delta W < 0$, and a "non-equilibrium" system if there are variations for which $\delta W > 0$. By definition, sub-equilibrium states are stable and non-equilibrium states are unstable, and the stability of the equilibrium states depends on the sign of $\delta(\delta W)$, where the second variation is computed by Griffiths' method. The equilibrium state of the "body with cracks-load" system is stable if, for all variations $\delta(\delta W) < 0$, and unstable if there are variations for which $\delta(\delta W) > 0$. The case where $\delta(\delta W) = 0$ for some variations and $\delta(\delta W) < 0$ for others corresponds to neutral (critical) equilibrium of the system or is problematical.

We will represent the virtual work in the form $\delta W = \delta W_e + \delta W_i + \delta W_f$, where δW_e is the work of the applied forces, δW_f is the work of the internal forces and δW_f is the work done in moving the crack tips. It is natural to introduce generalized forces by means of the relations

$$\delta W_e + \delta W_i = \sum_{j=1}^{m} G_j \delta a_j, \quad \delta W_f = -\sum_{j=1}^{m} \Gamma_j \delta a_j \tag{1.1}$$

We shall henceforth refer to the forces G_j as active (forces which propagate cracks), the forces Γ as passive (resistance forces). A fatigue crack will not grow if all $G_j < \Gamma_j$. When the equality $G_k = \Gamma_k$ is reached for one of the a_k, a crack can begin to grow. This growth will be stable if $\partial G_k/\partial a_k < \delta\Gamma_k/\partial a_k$ and unstable if $\partial G_k/\partial a_k > \delta\Gamma_k/\partial a_k$. If at least one of the relations between the generalized forces takes the form $G_k > \Gamma_k$, the system becomes unstable with respect to the corresponding generalized coordinate. A more general case is considered in [5–7].

Consider a one-parameter crack, that is, one whose behaviour is described by means of the one parameter of length a. Then the condition for growth to start, to continue and to arrest can be expressed in terms of the active generalized force G and the generalized resistance force Γ

$$G \lesseqgtr \Gamma \tag{1.2}$$

In the general case, both sides of (1.2) depend on the load parameters (for instance, on the extreme values of the applied stresses), the parameters of the environment (the concentration of the active agent, pH value and temperature), as well as on certain variables which characterize the distribution of damage at the crack tip and on its continuation. In the simplest case it is sufficient to introduce two scalar

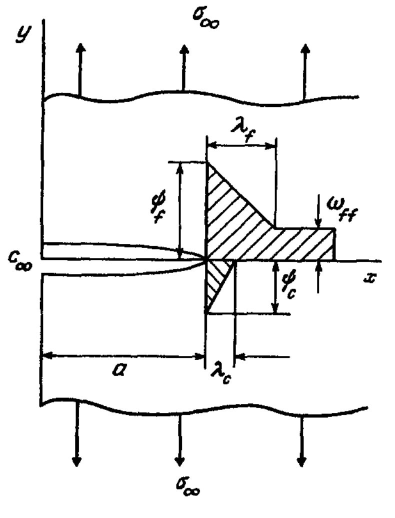

Fig. 1.

measures, one of which describes the mechanical component of microdamage and the other—the corrosion component. This means that relation (1.2) must be supplemented by the equations of damage accumulation over time.

2. THE MODEL OF A CORROSION FATIGUE CRACK

The proposed model is shown in Fig. 1. A body with a surface crack of depth a is subject to cyclic stresses σ_∞, in the direction of the normal to the plane of the crack, and an active agent, the properties of which at the crack mouth can be characterized by a certain parameter c_∞ (concentration, pH value, etc.). We distinguish two characteristic zones at the crack tip. The zone with scale λ_f has intensive accumulation of the mechanical component of fatigue damage. The level of damage is described by the scalar measure ω_f, where $0 \leqslant \omega_f \leqslant 1$, the value $\omega_f = 0$ corresponding to the undamaged material and $\omega_f = 1$ to the totally damaged material [8, 9]. We will denote the measure of mechanical damage at the crack tip by ψ_f, and the level of damage beyond the end of the crack (in the far field) by ψ_{ff}.

The end zone of corrosion damage is, in general, different from the end zone of mechanical damage. Confining ourselves to corrosion which is accompanied by the formation of an oxide film (Fig. 1), we will identify the dimensions of the end zone of corrosion damage with the film thickness λ_c. In the typical case $\lambda_c \ll \lambda_f$. We will introduce a measure of corrosion damage $0 \leqslant \omega_c \leqslant 1$, denoting its value at the crack tip by ψ_c.

In fatigue theory, cracks cannot be interpreted as mathematical cuts. For a complete description of the conditions at the crack tip, one must introduce either the effective radius of curvature of the crack at the tip δ (in the model of a thin plastic zone, for instance). It is assumed below that the material deforms elastically, and the radius ρ is taken as a characteristic describing the stress concentration at the crack tip when the fractographic picture is complicated and the material is less rigid near the tip.

Thus, under the joint action of cyclic stresses and the environment the behaviour of the material is characterized by the measures of damage ψ_f, ω_{ff} and ω_c, the size of the end zones λ_f, λ_c and the effective radius of curvature at the tip ρ. Generally speaking, the size of the end zones and the effective radius at the tip depend on the level of damage at the tip and the history of the crack propagation. In particular, when describing the spontaneous growth of a crack, we need to allow for both a change in ρ (blunting and sharpening of the tip) and the possibility that the oxide film fractures.

For further analysis, we need to formulate the equations of damage accumulation. The threshold-power law [1] is general enough for this. According to this law, we use following equations for the measures of damage on the continuation of the crack tip $x \geqslant a, y = 0$

$$\frac{\partial \omega_f}{\partial N} = \left(\frac{\Delta\sigma - \Delta\sigma_{th}}{\sigma_d}\right)^m, \quad \frac{\partial \omega_c}{\partial t} = \frac{1}{t_c}\left(\frac{c - c_{th}}{c_d}\right)^n \tag{2.1}$$

Here N is the number of cycles, taken as a continuous argument; t is the time; $\Delta\sigma$ is the tensile stress amplitude and c is the concentration of agent at the crack tip and its continuation. The following parameters of the material appear on the right-hand sides of Eqs (2.1): σ_d and c_d are characteristics of the material resistance to accumulated mechanical damage, $\Delta\sigma_{th}$, c_{th} are the threshold values of the resistance; m and n are positive indices and t_c is a time constant. If $\Delta\sigma < \Delta\sigma_{th}$ or $c < c_{th}$, the right-hand sides in the corresponding Eqs (2.1) must be equated to zero. To make things simpler, we shall neglect "crack closure" and related phenomena [10], assuming that $\sigma_\infty^{min} > \sigma_{cl}$, where σ_{cl} is the "crack closure" stress.

It is easy to obtain approximate equations for change in the damage measures at the tip of a moving crack. Replacing the partial derivatives in (2.1) by sub-stationary ones and noting that, within the end zones, $\partial\omega_f/\partial x \approx (\psi_f - \omega_{ff})/\lambda_f$, $\partial\omega_c/\partial x \approx \psi_c/\lambda_c$, we arrive at the equations

$$\frac{d\psi_f}{dN} + \frac{\psi_f - \omega_{ff}}{\lambda_f}\frac{da}{dN} = \left(\frac{\Delta\sigma_t - \Delta\sigma_{th}}{\sigma_d}\right)^m, \quad \frac{d\psi_c}{dt} + \frac{\psi_c}{\lambda_c}\frac{da}{dt} = \frac{1}{t_c}\left(\frac{c_t - c_{th}}{c_d}\right)^n \tag{2.2}$$

where the values of $\Delta\sigma_\tau$ and c_t are taken at the crack tip $x = a$. Equations (2.2) apply to both an immobile tip and a tip that is propagating relatively rapidly, when the first terms on the left-hand sides of the equations can be neglected.

The effective radius at the tip ρ appears implicitly in the first of Eqs (2.2) in terms of the stress amplitude $\Delta\sigma_\tau$. If the material remains linearly elastic and the change in the deformation properties at the crack tip is taken into account indirectly in terms of the effective radius ρ, then

$$\Delta\sigma_t \approx \Delta\sigma_\infty\left[1 + Z(a/\rho)^{1/2}\right] \tag{2.3}$$

where Z is a coefficient of the order of unity (for an elliptical gap $Z = 2$). When setting up an equation for ρ, we must take into account three interacting processes: sharpening of the tip as crack growth accelerates, blunting due to the accumulation of mechanical damage, and blunting due to corrosion damage. Since the initial value of the radius plays an important part at the initial stage of crack growth, there is a certain first-order differential equation that the function $\rho(N)$ must satisfy. In its simplest form it is

$$\frac{d\rho}{dN} = \frac{\rho_s - \rho}{\lambda_\rho}\frac{da}{dN} + (\rho_f - \rho)\frac{d\psi_f}{dN} + \frac{\rho_c - \rho}{f}\frac{d\psi_c}{dt} \tag{2.4}$$

The first term on the right-hand side describes sharpening up to the "acute" value of the radius ρ_s and the other two describe blunting up to "blunt" values ρ_f and ρ_c for mechanical and corrosion damage, respectively. Here $\rho_s \leqslant \min\{\rho_f, \rho_c\}$, and ρ_f and ρ_c can be of the same order. The length parameter λ_ρ appearing in (2.4) characterizes the distance which the crack tip must traverse in order that the sharpening effect becomes important. It is obvious that λ_ρ is of the same order as the size of the end zone λ_ϕ. In addition, Eq. (2.4) contains the frequency of the change of stresses f.

One more variable which must be discussed is the parameter of the active medium c. To fix our ideas, we shall refer to the concentration of active agent, or simply the concentration. For small cracks $c_t \approx c_\infty$, where c_∞ is the concentration at the body surface, that is, at the crack mouth. For deep cracks $c_t < c_\infty$. To find c_t, we must solve a hydrodynamic problem, allowing for diffusion of the agent and its interaction with the end zone, and with the lateral surfaces of the crack. The problem is complicated by the fact that the crack changes shape both as a result of growing and due to its partial closure. Moreover, actual fatigue cracks are irregular in shape and not straight. Although one could consider the corresponding hydrodynamic problem in a one-dimensional formulation [11], we shall use a phenomenological model for c_t similar to Eq. (2.4).

We will introduce the differential equation

$$\frac{dc_t}{dt} = \frac{c_a - c_t}{t_a} - \frac{c_t}{\lambda_c}\frac{da}{dt} \tag{2.5}$$

where c_a is the steady-state value of the concentration at the immobile tip, λ_c is a parameter of length like λ_ρ in Eq. (2.4) and t_a is a time parameter, which characterizes the rate of change of c_t with an arrested tip. This parameter obviously depends on the crack depth a and the frequency of loading f. We take

$$c_a = c_\infty\left[1 + \frac{a}{a_\infty}\left(1 + \frac{f}{f_\infty}\right)^{\alpha_f}\right]^{-\alpha_c}, \quad \alpha_f > 0, \quad \alpha_c > 0 \tag{2.6}$$

where a_∞ and f_∞ are parameters of the material having the dimensions of length and frequency respectively.

For slow loading processes $f \ll f_\infty$, and formula (2.6) gives $c_a = c_\infty[1 + (a / a_\infty)]^{-\alpha_c}$. As the crack gets deeper c_a decreases, owing to the fact that fresh agent has difficulty reaching it. If $f \gg f_\infty$, the mixing process intensifies, so that (other conditions remaining the same) a_a approaches c_∞. Thus, the model (2.5) and (2.6) is sufficiently flexible to describe the change of concentration at the tip with allowance for the main factors: the depth of the crack, its growth rate and the frequency of loading.

3. THE EQUATIONS OF CRACK GROWTH

The relation between the generalized forces for a one-parameter fatigue crack has the form

$$G \lesseqgtr \Gamma \tag{3.1}$$

where, for the initial crack, $G < \Gamma$. For a crack which is growing in stable fashion, $G = \Gamma$, $\partial G/\partial a < \partial\Gamma/\partial a$. Since the active generalized force G characterizes the energy liberation in the entire "body with crack-load" system, the influence of microdamage on the value of this force is quite insignificant [12]. Thus we can put

$$G = K^2(1 - \nu^2)/E \tag{3.2}$$

where K is the stress intensity factor (here and below any reference to the mode of fracture is omitted), E is Young's modulus and ν is Poisson's ratio of the material. The value of G in (3.1) is taken at a time at which the applied stresses reach maximum values $\sigma_\infty^{\max}(N)$. Then in formula (3.2)

$$K_{\max} = Y\sigma_\infty^{\max}(\pi a)^{1/2}, \quad Y = O\ (1) \tag{3.3}$$

When calculating the generalized resistance force Γ, we need to allow for both the mechanical and the corrosion components. The force Γ can be expressed in terms of the specific work of fracture γ, which is equal to the energy which must be expended to advance the crack tip over unit area. There is, as yet, no reliable experimental data on the effect of microdamage on the characteristics of crack stability. The simplest model is an additive one [13]; here

$$\gamma = \gamma_0[1 - (\omega_f + \omega_c)^\beta / \omega_r^\beta] \tag{3.4}$$

where γ_0 is the specific work of fracture for the undamaged material and $\beta > 0$. Suppose that ω_f in formula (3.4) can take values greater than unity. When $\omega_r > 1$, the value $\gamma_0(1 - \omega_r^{-\beta})$ characterizes the residual crack stability of the damaged material, that is, material for which $\omega = \omega_f + \omega_c = 1$. Formula (3.4) corresponds to the expression for the generalized resistance force

$$\Gamma = \gamma_0[1 - (\psi_f + \psi_c)^\beta / \psi_r^\beta] \tag{3.5}$$

An approximate equation for the steady growth of a corrosion fatigue crack is obtained from the approximate equations (2.2), neglecting the first terms on their left-hand sides. Then

$$\psi_f \approx \omega_{ff} + \lambda_f\left(\frac{da}{dN}\right)^{-1}\left(\frac{\Delta\sigma_t - \Delta\sigma_{th}}{\sigma_d}\right)^m, \quad \psi_c \approx \frac{\lambda_c}{f t_c}\left(\frac{da}{dN}\right)^{-1}\left(\frac{c_t - c_{th}}{c_d}\right)^n \tag{3.6}$$

Substituting (3.6) into (3.5) and equating the value of G in (3.2) to Γ, we obtain an equation which is solved for da/dN

$$\frac{da}{dN} = \left[\lambda_f\left(\frac{\Delta\sigma_t - \Delta\sigma_{th}}{\sigma_d}\right)^m + \frac{\lambda_c}{f t_c}\left(\frac{c_t - c_{th}}{c_d}\right)^n\right]\left[\omega\left(1 - \frac{K_{\max}^2}{K_c^2}\right)^{1/2} - \omega_{ff}\right]^{-1} \tag{3.7}$$

$$K_C^2 = \gamma_0 E/(1 - \nu^2)$$

where we have introduced the notation K_C for the critical stress intensity factor ("critical" as in fracture mechanics).

The expression in the first square brackets on the right-hand side of Eq. (3.7) allows for the contribution of microdamage at the tip and that in the second square brackets allows for the contribution of the energy balance in the "body with crack-load" system and also the far-field microdamage. The crack growth becomes unstable when $K_{\max} = K_{fc}$, where K_{fc} is the critical stress intensity factor for fatigue cracks

$$K_{fc} = K_c[1 - (\omega_{ff} / \omega_r)]^{\beta/2} \tag{3.8}$$

Formula (3.8) takes into account the lower crack stability due to microdamage accumulated in the far field, that is, before the crack tip reaches a position at which actual fracture occurs. For $K \ll K_{fc}$ we have the equation

$$\frac{da}{dN} = \left[\lambda_f\left(\frac{\Delta\sigma_t - \Delta\sigma_{th}}{\sigma_d}\right)^m + \frac{\lambda_c}{f t_c}\left(\frac{c_t - c_{th}}{c_d}\right)^n\right] \tag{3.9}$$

Formally, the right-hand side of Eq. (3.9) looks like the result of superposing the rates associated with mechanical and corrosion damage [4]. In reality, however, even such a simple equation describes the interaction of all the processes. Thus, the stress amplitude $\Delta\sigma_t$ depends on the stress concentration at the tip which, in turn, depends on both damage measures; the concentration of agent c_t at the tip depends on the depth of the crack and the rate at which it is growing, etc. A further complication arises from the fact that the size of the end zones is, generally speaking, not a constant of the material. The value of λ_f is of the order of a few radii of curvature at the tip or, in the model of a thin plastic zone, the length of that zone [2]. The value of λ_c can be associated with the measure of corrosion damage, putting $\lambda_c = \lambda_0\psi^\alpha$, for example, where λ_0 corresponds to a fully formed oxide film, $\alpha > 0$. On the whole, Eqs (3.7) and (3.9) are more illustrative in nature, demonstrating that the conclusions of the theory can be interpreted in the context of known semi-empirical equations [3, 14]. If no further simplifications are made, the only way of using the proposed model is to perform a computational experiment.

4. NUMERICAL MODELLING

The computational algorithm contains the simultaneous solution of Eqs (2.4)–(2.5) before the first equilibrium state is reached. At that instant we have

$$K_{\max}^2 = \frac{E\gamma_0}{1-\nu^2}\left[1-\left(\frac{\psi_f+\psi_c}{\omega_r}\right)^\beta\right] \tag{4.1}$$

After the first equilibrium state is reached, the size of the crack is given a small increment Δa, after which the computation cycle is repeated. The algorithm additionally includes internal iteration loops, since the variables $\sigma_t(N)$, $\omega_f(x, N)$, $\omega_c(x, N)$, $\rho(N)$ and $c_t(N)$ are mutually conditioned. For the stresses $\sigma(x)$ on the continuation of the crack tip, we use the solution for an elliptical crack

$$\frac{\sigma}{\sigma_\infty} = \frac{\xi^2+\mu}{\xi^2-\mu} + \frac{(1-\mu)^2[\xi^4+3\xi^2+\mu(\xi^2-1)]}{2(\xi^2-\mu)^3} \tag{4.2}$$

$$\xi = \frac{(x/a)+[(x/a)^2+(\rho/a)^2-1]^{1/2}}{1+(\rho/a)^{1/2}}, \quad \mu = \frac{1-(\rho/a)^{1/2}}{1+(\rho/a)^{1/2}}$$

The use of formulae (4.2) removes the need to find the size of the end zone λ_f. The thickness of the corrosion film is taken as $\lambda_c = \lambda_0\psi_f$.

We used the following basic numeral data: E = 300 GPa, ν = 0.3 and γ_0 = 10 kJ/m^2. In the first equation of (2.1) we took σ_d = 5 GPa, $\Delta\sigma_{th}$ = 250 MPa with a coefficient of asymmetry of the cycle $R = \sigma_\infty^{\min}/\sigma_\infty^{\max} = 0$ and exponent m = 4. In the second equation of (2.2), the concentration c is relative to the parameter of the material c_d, so that the conditions of the surrounding medium are expressed in terms of the dimensionless quantity c_∞/c_d, where c_∞ is the concentration at the crack tip. The threshold concentration was taken as c_{th} = 0 and the exponent n = 4. We took the following values for the characteristic lengths in Eqs (2.4) and (2.5): ρ_s = 10 μm, $\rho_f = \rho_c = \lambda_\rho = \lambda_c$ = 100 μm. The maximum thickness of the oxide films λ_0 = 10 μm. The constant times in the second equation of (2.1) and Eq. (2.5) were taken as $t_c = t_a$ = 1 h. In formula (2.6) a_∞ = 1 μm, f_∞ = 1.

The results of the computational experiment for $\Delta\sigma_\infty$ = 100 MPa and f = 1 Hz are shown in Figs 2–4. The initial data were: a_0 = 1 μm, ρ_0 = 50 μm; all the damage parameters are zero in the initial state. Curves 1–4 in Figs 2 and 4 correspond to concentration levels c_∞/c_d = 0.25; 0.5; 0.75; 1.0. The change in concentration at the tip c/c_d as a function of the number of cycles N is shown in Fig. 2(a). As the crack deepens, its growth rate increases, and the concentration of the active agent at the tip declines, falling rapidly before final fracture.

Figure 2(b) shows the change in the effective radius of curvature ρ during the crack growth. The stage crack sharpening from the initial value ρ_0 = 50 μm to a value close to ρ = 80 μm cannot be seen. Steady growth proceeds at a radius close to this value. Sharpening of the tip starts at the final stage of crack growth. However, for high values of c_∞/c_d, some sharpening is observed at the initial stage. This can be explained by the interaction of mechanical and corrosion damage.

Figure 3 shows the behaviour of the measures of microdamage ψ_f, ψ_c and $\psi = \psi_f + \psi_c$ for c_∞/c_d = 0.25 (a) and c_∞/c_d = 1 (b). The graphs illustrate the interaction between the different mechanisms and the way in which the contribution of each damage component changes during crack growth. For small c_∞/c_d mechanical damage predominates, so that $\psi \approx \psi_f$. At the initial stage of crack growth a pure corrosion component ψ_c is also observed. Near ψ = 1 the crack moves away. For large c_∞/c_d the contribution made by corrosion damage is substantial (Fig. 3b). As the crack tip moves, the corrosion component decreases and the mechanical component increases. Both components (like their sum) start to decrease rapidly as final fracture approaches. The quantity ψ_c can be interpreted as the dimensionless thickness of the oxide film. By the end of the process, this quantity falls practically to zero. The most important feature of the graph of Fig. 3(b) is the spasmodic behaviour of the measures ψ_f and ψ_c at the initial stage. This can be interpreted as the result of alternate sharpening and blunting of the crack.

The crack size a as a function of the number of cycles N is shown in Fig. 4(a).

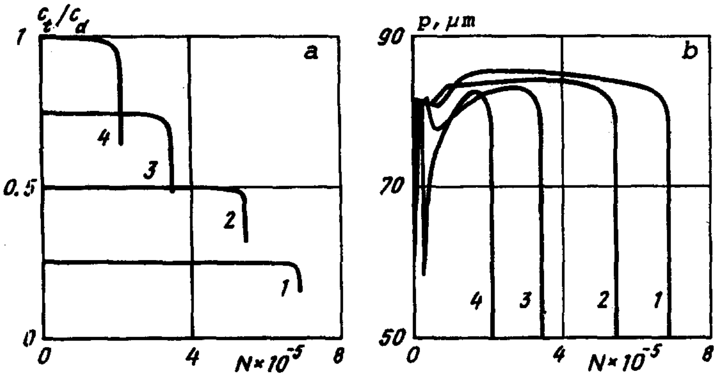

Fig. 2.

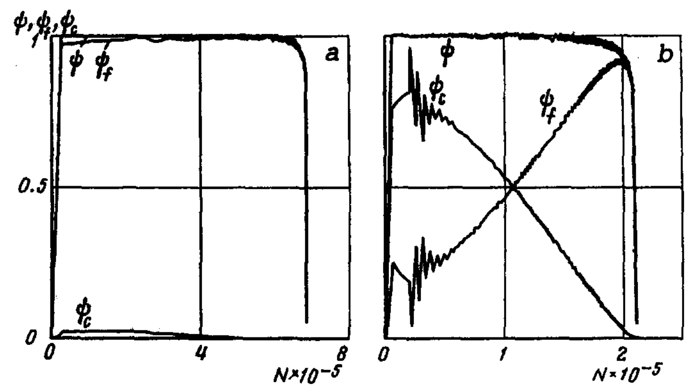

Fig. 3.

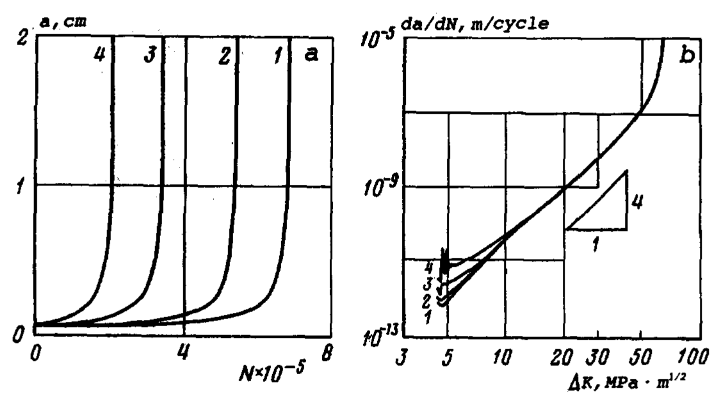

Fig. 4.

Figure 4(b) shows the crack growth in standard form (the growth rate da/dN as a function of the amplitude of the stress intensity factor ΔK). The first part of the curves differs from the usual experimental diagrams of the fatigue crack growth. In particular, there is a non-monotonic dependence of the growth rate da/dN as a function of the amplitude ΔK and considerable scatter, depending on the concentration of the active agent. At high concentration, the initial part of the diagram can be interpreted as a "plateau", corresponding to the low contribution of the purely mechanical component to the crack propagation rate. The mechanical component then starts to predominate. The middle part of the diagram corresponds to the usual Paris–Erdogan approximation with angular coefficient close to $m = 4$. Crack growth then accelerates until final fracture.

Figure 5, with $c_\alpha/c_d = 1$, illustrates the influence of the load frequency on the crack growth rate. Curves 1–4 are for frequencies $f = 0.1$, 1.5 and 10 Hz, respectively. The velocity is measured in m/cycle in Fig.

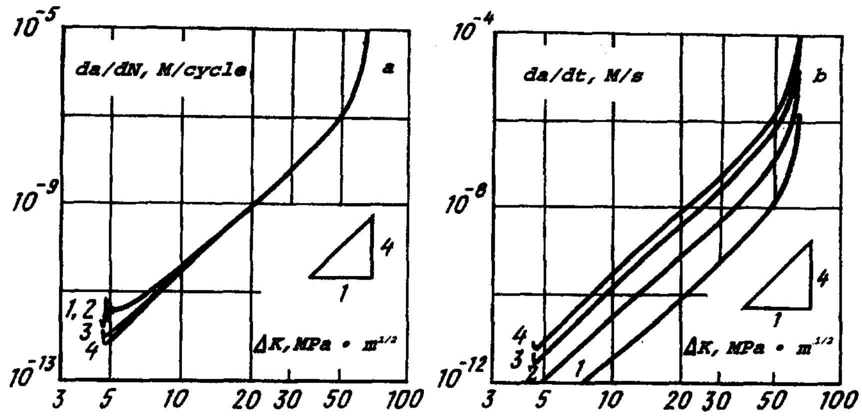

Fig. 5.

5(a), and in m/s in Fig. 5(b). The greatest divergence in the first diagram is on the initial segments of the curves, where corrosion damage predominates. The divergence is substantial throughout the whole of the second diagram. In both cases the angular coefficient in the middle part of the curves is close to the exponent $m = 4$ in Eq. (2.1).

These graphs represent only a small part of the computational experiment performed, in which a study was made of the combined influence of extreme cyclic stresses, the µm loading frequency and the initial conditions on the crack growth rate and the behaviour of the other defining parameters: the concentration of the active agent, effective radius at the tip, and measures of mechanical, corrosion and total damage. We have shown that some of the parameters of the proposed model can be determined from the results of standard tests of corrosion fatigue and, in particular, from experimental diagrams of crack growth for different levels of cyclic stresses, different concentrations and different initial conditions.

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